angles observed at heated-rail temperatures of 1700°C and 2200°C.[1]

The type 2 sawtooth column is associated with high arc Mach numbers. Though high Mach number arcs were observed which seemed not to have the type 2 sawtooth structure, only one type 2 arc was observed with a Mach number as low as 3.6, and all other type 2 arcs moved at Mach numbers above 4.5.

The appearance of the type 2 sawtooth at high Mach numbers could have one important implication. It could mean that the electric field plays a more important role in column slanting at high Mach numbers, since, as was noted earlier, the electric field can be important to the (equal) slanting of the segments of the type 2 column. This implication is supported by the fact that the peak in the ionization parameter is much sharper for $M \geqq 4.5$. However, these observations cannot be regarded as conclusive proof of an electric-field or ionization mechanism for slanting until more detailed studies of the column structure can be made. Changes in the nature of the finer structure could result from mutual interaction between adjacent sawtooth segments.

Conclusions

The following conclusions may be drawn: 1) There is a fine structure for the supersonic electric arc in sulfur hexafluoride. Side-view photographs show this structure to be such that the electric current of the moving arc column describes a slanted sawtooth path between electrodes. Front-view photographs show that the column axis does not spiral (generally) but remains in the electrode plane. Plasma streamers are sometimes visible in the wake of the sawtooth column, each streamer originating at a downstream apex of the sawtooth shape. 2) The sawtooth structure of the arc column in SF_6 results in an apparent crossflow Mach number which can be significantly higher than the actual crossflow Mach number for a local column segment. 3) Measurements indicate that for arc Mach numbers from about 1.5 to about 5.5, the actual crossflow Mach number is very nearly equal to unity. 4) Using an actual crossflow Mach number of unity, good agreement is obtained between the measured angle of slant of the arc column and that calculated from measurements of the sawtooth angle using the sawtooth-column theory. 5) The experimental observations indicate that the convective interaction mechanism which has been previously observed for the electric arc in air, and which results in a stable arc column slanted across the electric field lines such that the crossflow Mach number is near unity—the observations indicate that this mechanism is also locally effective for the electric arc in SF_6, and that in cases where transient conditions or root constraints dictate an arc with less slant, the arc column in SF_6 takes on a fine structure which nevertheless maintains the actual local crossflow Mach number near unity. 6) The apparent transition from the type 1 to the type 2 sawtooth at higher Mach numbers gives evidence of an ionization mechanism for column slanting, and indicates that there may in fact be two mechanisms for slanting, each of which tends to keep the actual crossflow Mach number near unity.

References

[1] Bond, C. E. and Wickersheim, D. N., "Convective Electric Arcs at Mach Numbers up to 6.5," *AIAA Journal*, Vol. 8, No. 10, Oct. 1970, p. 1748.

[2] Bond, C. E., "Slanting of a Magnetically Stabilized Electric Arc in Supersonic Flow," *The Physics of Fluids*, Vol. 9, No. 4, April, 1966, p. 705.

[3] Bond, C. E. and Potillo, R. W., "Stability and Slanting of the Convective Electric Arc in a Thermionic Rail Accelerator," *AIAA Journal*, Vol. 6, No. 8, Aug. 1968, p. 1965.

[4] Ahearn, A. G. and Hannay, N. B., "The Formation of Negative Ions of Sulfur Hexafluoride," *Journal of Chemical Physics*, Vol. 21, No. 1, Jan. 1953, p. 119.

[5] Schumb, W. C., Trump, J. G., and Priest, G. L., "Effect of High Voltage Electrical Discharges on Sulfur Hexafluoride," *Industrial and Engineering Chemistry*, Vol. 41, No. 7, 1949, p. 1348.

Mach Disk in Underexpanded Exhaust Plumes

MICHAEL ABBETT*
Aerotherm Corporation, Mountain View, Calif.

Nomenclature

M = Mach number
p = pressure
r_j = jet exhaust radius
x = abscissa
y = ordinate (normal to centerline)
δ = streamtube width
γ = ratio of specific heats
ρ = density
τ = streamline slope = $\tan\theta$
θ = angle streamline makes with axis

Subscripts

SS = slipstream separating subsonic core streamtube from supersonic outer flow downstream of Mach disk
TP = triple point
j = conditions at jet exhaust plane
∞ = ambient conditions

THE basic features of the inviscid supersonic plume for static ambient are shown in Fig. 1. The expansion waves from the nozzle lip reflect from the constant pressure streamline as compression waves, subsequently coalescing to form the intercepting (barrel) shock. Depending on the flow conditions, the intercepting shock may reflect regularly at the centerline or it may terminate in a triple point-Mach disk configuration, illustrated in Fig. 1. Behind the Mach disk is a region of subsonic flow bounded above by a slipstream emanating from the triple point. This Note presents a flow model which explains in detail why a Mach disk is formed from the plume intercepting shock and, when it is formed,

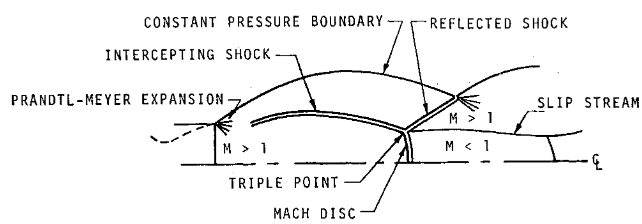

Fig. 1 Inviscid plume of underexpanded nozzle—static ambient.

Presented as Paper 70-231 at the AIAA 8th Aerospace Sciences Meeting, New York, January 19–21, 1970; submitted April 20, 1970; revision received September 28, 1970. This Note is a condensed version of AIAA Paper 70-231, which included discussions of and comparisons with other theories. Research was supported by NASA Manned Space Flight Center.

* Staff Engineer. Member AIAA. This work was completed while author was employed at General Applied Science Laboratories, Westbury, N.Y.

where the triple point is located. Quantitative results obtained verify the theory.

In an underexpanded plume, the favorable axial pressure gradient resulting from the expansion at the nozzle lip gradually diminishes in strength downstream. In some downstream region the compression dominates the expansion, leading to the adverse pressure gradient necessary to bring the flow near the axis up to ambient pressure. The integrated effect of these compressions often results in the adverse pressure gradient being very large so that the pressure increase is concentrated over short distances.

A model of the recompression process can be constructed by viewing the flow as divided into two parts, viz., a quasi-one-dimensional streamtube along the centerline and the rest of the flow. In the favorable axial pressure gradient the cross-sectional area of the centerline streamtube increases axially, the amount being determined by the interaction of the streamtube with the outer flow. Since it is supersonic, the streamtube is supercritical and $d\delta/dp < 0$ [δ is the width (radius) of the streamtube]. No problem is associated with the interaction so long as $dp/dx < 0$. However, as Crocco[2] points out, the supercritical centerline flow, interacting with the supersonic outer flow, is not able to smoothly generate an adverse axial pressure gradient. Instead, the supercritical core flow reacts to the required downstream pressure increase by jumping to a subcritical (subsonic) state. If regular reflection of the intercepting shock from the axis results in a large enough pressure increase so that significant additional recompression is not required downstream of the reflection point, then a strong shock is not necessary and there will be no Mach disk. If, however, regular reflection achieves only part of the required pressure increase, then the inability of the supersonic core streamtube to react with the supersonic outer flow to generate the additional adverse pressure gradient will lead to the formation of a Mach disk, bringing the core flow to subsonic velocities.

The triple point Mach disk location reflects quantitatively the interaction between the subsonic core and supersonic outer flow downstream of the Mach disk. Specifically, the triple point location is determined by the requirement that the centerline core flow, which is subsonic just downstream of the Mach disk, must pass smoothly through a throat-like region where the flow becomes supersonic. In the quasi-one-dimensional (streamtube) approximation for the core flow, the streamtube flow will be sonic and its cross-sectional area will have a minimum in the throat-like region.

The abscissa of the triple point can thus be considered to be a parameter of the inviscid solution for the plume flowfield. By assuming a triple point abscissa, the triple point-Mach disc solution can be obtained and used as initial conditions for a solution of the interaction problem downstream. In general, the resulting initial value problem will not possess

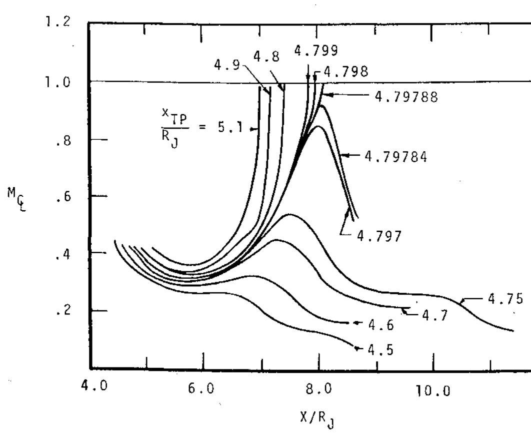

Fig. 2 Core streamtube Mach number vs axial distance for various assumed triple point abscissa.

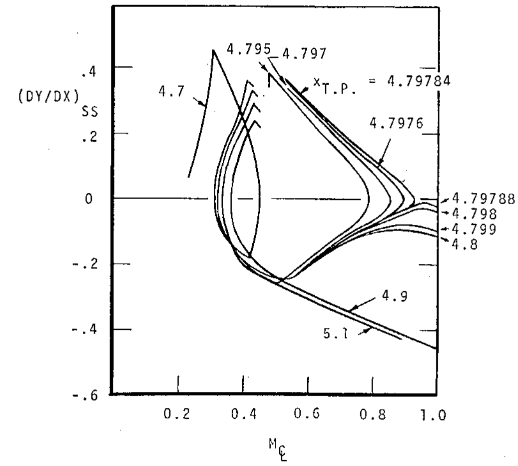

Fig. 3 Slip stream slope vs core Mach number for various assumed values of triple point abscissa.

a consistent smooth solution. The pressure gradient generated will tend to either $\pm\infty$, with $M_{℄} \rightarrow {0 \atop 1}$ simultaneously with $(dy/dx)_{SS} \rightarrow \pm\infty$. Only for one value of the triple point location will $(dp/dx)_{℄}$ remain finite while $M_{℄} \rightarrow 1$ as $(dy/dx)_{SS} \rightarrow 0$ in a throat-like region. Thus, the throat-like region is a saddle point type singularity depending on the triple point location as a parameter.

The preceding discussion has perhaps overplayed the role of the intercepting shock, not in the frequency of its appearance, but in its position in the fluid mechanical processes. The important item is obviously the adverse axial pressure gradient necessary to bring the core flow back up to the ambient pressure level. From this standpoint, it is perfectly admissible that there be no intercepting shock at all.

Verification of the Model

The part of the theory locating the Mach disk was verified by a computational example which illustrates quite well the saddle point character of the solution in the throat region and the eigenvalue character of the theory. A series of computations have been made, each having a different value for x_{TP}, the triple point abscissa. An analysis of the behavior of the streamtube solution downstream of x_{TP} as a function of x_{TP} comprises the quantitative support of the theory. The conditions for this example are 1) perfect gas, inviscid, constant specific heat ratio, $\gamma = 1.4$; 2) static ambient; 3) underexpansion ratio, $p_j/p_\infty = 4.0$, $M_j = 1.5$, and axisymmetric, uniform parallel exhaust. This case, one of those reported by Love[3] et al., has a Mach disk diameter comparable to the nozzle exit diameter, so the interaction should be strong enough and on a large enough scale so that numerics will not cloud the issue.

For orientation purposes only, the computational procedure is outlined here. The supersonic portions of the flowfield are computed by the method of characteristics. The intercepting shock is detected when two right running characteristics coalesce, the determination of its shape and strength being subsequently part of the flowfield computation. When the abscissa of the intercepting shock reaches the value specified for the triple point, a triple point solution is generated by matching pressure and flow angle behind the strong and reflected shocks. Then the computation continues, including the solution of the subsonic quasi-one-dimensional flow downstream of the Mach disk. The centerline pressure gradient and the shape of the slipstream downstream of the triple point are determined by matching the pressure and flow angle along the slipstream.

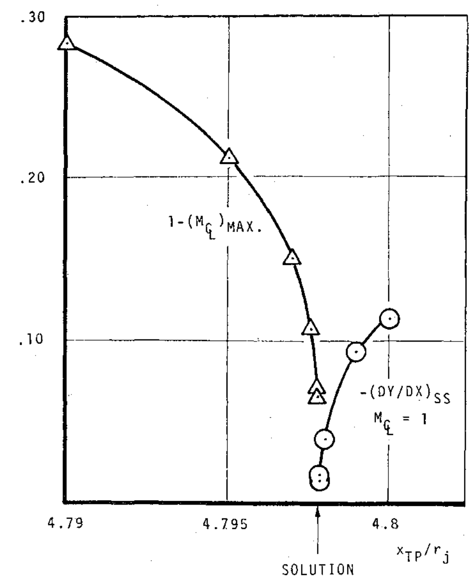

Fig. 4. Parameters pinpointing the solution.

As the computation proceeds downstream of the triple point, the solution for the subsonic streamtube will follow either an accelerating or a decelerating branch, depending on whether the assumed value of x_{TP} is too small or too large. Near the correct triple point abscissa neither case quickly prevails, and the current "branch" does not become evident until the pressure gradient actually begins to increase or decrease catastrophically (in the first case this is evident by the catastrophic increase in θ_{SS}).

For the case computed, the axial variation in core streamtube Mach number (Fig. 2) clearly shows the two branches, depending on whether the assumed value for x_{TP} is greater than or less than the solution. In particular, notice the "cusping" of the stagnating branch as x_{TP} approaches the solution.

One of the most revealing figures is a plot of $(dy/dx)_{SS}$ vs $M_{℄}$ (Fig. 3). The saddle point behavior is quite evident, particularly in the tendency of $d\theta(M)/dM$ to become discontinuous at $\theta = 0$ as $M = 1$ is approached on the stagnating branch. Notice the convergent-divergent character of that branch when the figure is rotated 90° clockwise. It is clear from these two figures that, within the accuracy of these computations, this theory brackets x_{TP}/r_j between 4.79784 and 4.79788. This is in good agreement with the experimental result of Love et al.[3] which placed the Mach disc on the axis at $x/r_j = 4.8 - 4.9$.†

In closing, it is of interest to consider how close each iteration comes to passing through the "throat" for each assumed value of the triple point location. On the stagnating branch, the quantity $1 - (M_{℄})_{\max}$ measures closeness well, while on the accelerating branch, the value of the slip stream slope, $(dy/dx)_{SS}$, where $M_{℄} = 1$ is appropriate. These two curves strikingly point to the value of x_{TP} (Fig. 4).

References

[1] Crocco, L., "One-Dimensional Treatment of Steady Gas Dynamics," *High Speed Aerodynamics and Jet Propulsion, Vol. III, Fundamentals of Gas Dynamics*, edited by H. W. Emmons, Princeton Univ., Princeton, N.J., 1958.

[2] Crocco, L., "Considerations on the Shock-Boundary Layer Interaction," *High Speed Aeronautics*, edited by A. Ferri, N. J. Hoff, and P. Libby, Polytechnic Institute of Brooklyn, 1955.

[3] Love, E. S. et al., "Experimental and Theoretical Studies of Axisymmetric Free Jets," R6, 1959, NACA.

† Note that Love[3] measured the Mach disk abscissa, which is slightly downstream of x_{TP}.

Forces on an Inclined Circular Cylinder in Supercritical Flow

W. J. Bootle*
Avco Systems Division, Wilmington, Mass.

Nomenclature

C_D = cylinder drag coefficient based on planform area
C_{D_e} = pressure drag of elliptic section normal to the flow
C_L = cylinder lift coefficient based on planform area
C_N = cylinder normal force coefficient based on planform area
C_f = friction coefficient based on wetted area
D = drag
N = normal force
Re = Reynolds number
S = planform area (dl)
S_e = frontal area of elliptic section $(dl \sin\alpha)$
V = flow velocity
c = major axis of elliptic section
d = cylinder diameter
h = height of elliptical section of an element of cylinder
l = length of an element of an infinite cylinder
q = dynamic pressure
t = minor axis of elliptic section
α = angle of attack
ν = kinematic viscosity

Subscripts

∞ = based on freestream conditions
c = based on cross flow conditions
turb = turbulent flow conditions

THE classical method for estimating the aerodynamic forces acting on an inclined circular cylinder in low-speed flow is based on the cross-flow principle wherein the flow component normal to the body is treated as independent of that along it. In terms of freestream dynamic pressure and planform area, the normal force coefficient then becomes $C_{D_c} \sin^2\alpha$, where C_{D_c} is the section drag coefficient for the unyawed cylinder, and the axial force component due to friction becomes $\pi C_f \cos^2\alpha$; in turn the following expressions are obtained for C_L and C_D:

$$C_L = C_{D_c} \sin^2\alpha \cos\alpha - \pi C_f \cos^2\alpha \sin\alpha \tag{1}$$

$$C_D = C_{D_c} \sin^3\alpha + \pi C_f \cos^3\alpha \tag{2}$$

The method is known to give good correlation with test data in sub-critical flow, but to fail[1,2] in supercritical flow at cross flow Mach numbers below 0.5. During transition there is a reduction in C_{D_c} which for smooth cylinders ranges from a sub-

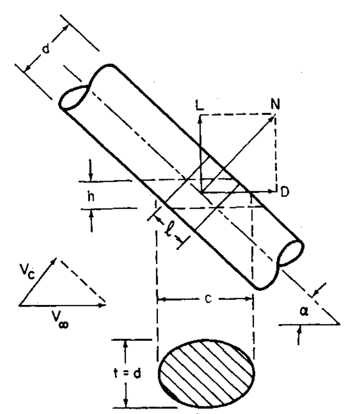

Fig. 1 Schematic representation of the analytical model.

Received March 5, 1970; revision received October 26, 1970. The author wishes to express his appreciation to E. E. H. Schurmann, Chief, Aerodynamics Section, Engineering Directorate, for his helpful suggestions during a review of the paper.

* Group Leader, Vehicle Dynamics, Flight Mechanics Section, Engineering Directorate. Member AIAA.